Copy the following conjecture and vocabulary in your notes:

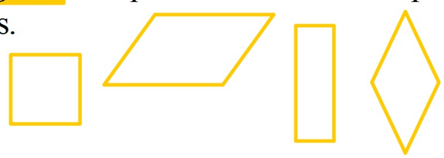
Conjecture 1.2 *Midline Conjecture*

A segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half its length.

congruent sides are sides that are equal in length.

congruent angles are angles that are equal in measure.

a **parallelogram** is a quadrilateral with two pairs of parallel sides.



1.10 Getting Started

Objectives:

- **Warm up to the ideas of the investigation.**
- **Search for numerical invariants, such as constant sums, products, differences, or ratios.**

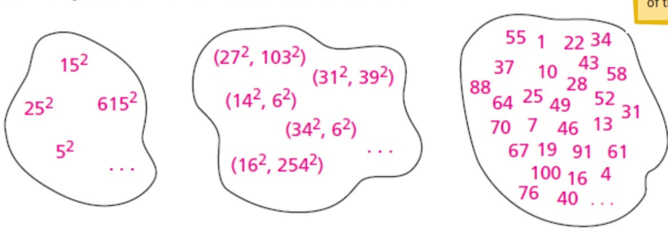
Something that is true for each member of a collection is an **invariant** for the collection.

Remember...

An invariant over a set is something that is the same for every member of the set.

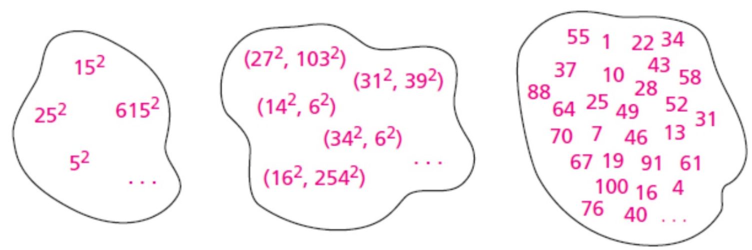
For You to Explore

1. Study the three different collections below.



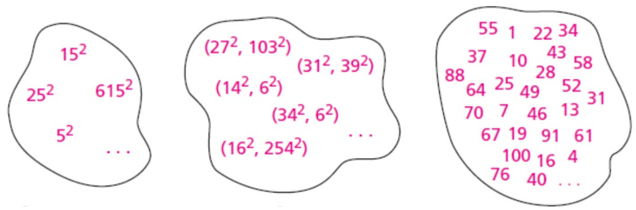
a. In the first set, each number before squaring ends in 5. Evaluate the squares. What invariants do you find?

All of the squares end in 25.



b. The second set contains pairs of square numbers. In each pair, the two numbers before squaring end in digits that add to 10. Evaluate all the squares. What invariants do you find?

If the units digits of a pair of whole numbers have a sum of 10, then the squares of the numbers have the same units digits.

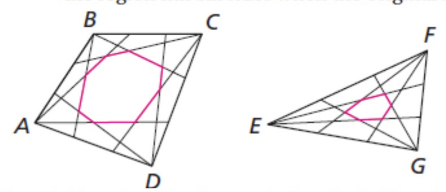


c. The third set contains 1, 4, 7, 10, 13, and so on. Explain whether 301 is in the set. Choose pairs of numbers from the set and find their products. What seems to be true about their products? What seems to be true about the sums of four numbers chosen from the set? What seems to be true about the differences of any two numbers?

Yes; the product of any two numbers in the set is also in the set; the sum of any four numbers in the set is also in the set; the difference of two numbers in the set is 1 less than some number in the set.

2. Draw two polygons like quadrilateral $ABCD$ and $\triangle EFG$ below. On each side, draw two points that roughly divide the sides into thirds. Connect each vertex to two points on different sides to form the largest angle possible. The connecting segments surround a region. The two diagrams here suggest

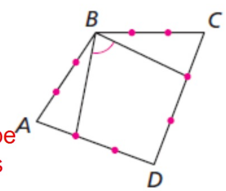
- the region has eight sides when the original shape has four sides
- the region has six sides when the original shape has three sides



Is this a reliable pattern? In other words, does the inside region always have twice the number of sides as the original shape when you connect vertices to "third points" in this way? Explain.

The triangle seems always to produce a hexagon, but the quadrilateral does not always produce an octagon. This can be verified by dragging the vertices of the quadrilateral to various positions

To make the angle with vertex at B, connect B to the point on $\overline{AD}$ that is closer to A, and to the point on $\overline{CD}$ that is closer to C.



On Your Own

Practice problems p.50 (3,4)

3. The table at the right shows pairs of numbers. An invariant for the table is 4 because $\frac{r}{q} = 4$ for each pair. Make three different tables like the one at the right so that the invariant for each table is 8. Use a different operation to build each table.

q	r
$\frac{1}{8}$	$\frac{1}{2}$
4	16
8	32
100	400

4. Draw a quadrilateral. Construct the midpoints of its sides. Then connect the consecutive midpoints.

a. Explain why the figure formed must be a quadrilateral.

b. Can the figure formed be *any* kind of quadrilateral, or are certain kinds of quadrilaterals not possible? Explain.