LAUNCH:

Why do you think that mathematicians need to prove statements?

How do you know the length of a segment or the measure of an angle? One way is to measure using a ruler or protractor, respectively. But measurement has some drawbacks.

- Measurement is not exact. No matter how precise your ruler or protractor is, neither will provide an exact measurement.

- Certain measurements are difficult or impossible to determine. For instance, the distance between two cities is difficult to find directly. The distance between Earth and the moon is impossible to measure directly.

- A measurement is only reliable if it holds true for an infinite number of cases.

One reason to prove results in geometry is to check measurements. For example, if you know from a theorem that two segments should be equal, then the measurements of those segments should be the same. Measurements are subject to error in a way that logical deduction is not.

2.6 Deduction and Proof

Objective: Students will make assumptions and write proofs to understand the need for proof in mathematics.

Go Trojans

Minds in Action   episode 3

Ivan: Sasha:

Sasha and Ivan are making triangular pennants for the school's sports teams. Ivan made a pennant with two 14-in. sides and a 30° angle between those two sides.

Ivan   I measured the other two angles of my triangle. One is 73 degrees and the other is 77 degrees.

Sasha   But they should both be the same. Measure more carefully.

So Ivan did.

Ivan   This time, they are 74.3 degrees and 75.7 degrees.

Sasha   Close. But they should both be the same.

Ivan   What do you mean? How do you know they should both be the same? My protractor says they are a little off. The bottom one is always a little bigger than the top one.

Sasha   They should both be the same. Look. Draw a median. The two triangles are congruent.

Ivan   Okay, I see. Yeah.

Sasha   And the two angles you are measuring are corresponding angles in the congruent triangles. So they have to be congruent.

Ivan   I guess I'll get a better protractor.

1. Ivan agrees that the two triangles formed by Sasha's median are congruent. Do you? Explain.

2. How do you know that the two angles that Ivan measured are corresponding parts of the two triangles formed by the median?
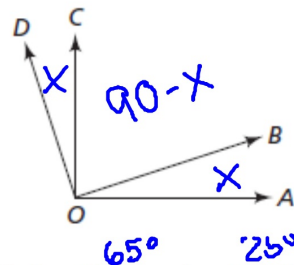
SSS

14 in

30°

14 in
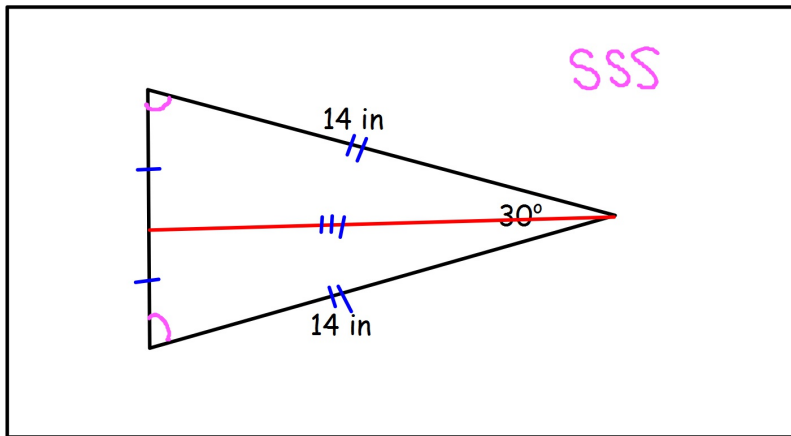
Corresponding
Parts of
Congruent
Triangles are
Congruent

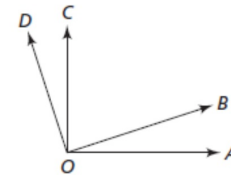* Corresponding parts of congruent triangles are congruent (CPCTC) *

EXAMPLES

1. Use the figure below. ∠COA and ∠DOB are right angles.

X   90-X

X

65°   25°

a. If m∠BOA = 25°, find m∠COB and m∠COD.
b. If m∠COB = 63°, find m∠BOA and m∠COD.
c. If m∠DOC = 31°, find m∠COB and ∠BOA.
d. If m∠DOC = 31°, find m∠DOA.
e. If m∠AOB = x°, find m∠COB and m∠COD.

TWO-COLUMN PROOF

2. Use the figure below. ∠COA and ∠DOB are right angles. Prove that m∠BOA = m∠COD.

Proof:

Statements	Reasons
m∠COA and m∠DOB = 90°	Given
m∠BOA + m∠COB = 90° m∠COD + m∠COB = 90°	Addition of the parts of the angle equal the total angle
m∠BOA = m∠COD	Basic rules of algebra

3. Standardized Test Prep $\overline{BE}$ bisects $\overline{AD}$ at C. Point C is the midpoint of $\overline{BE}$. Fill in the missing reason in the following proof.
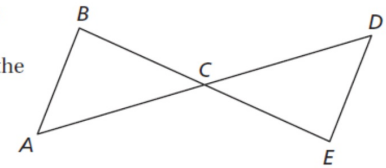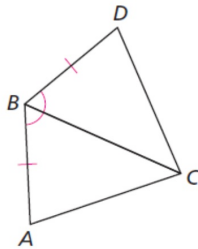

Statements	Reasons
1. $\overline{BE}$ bisects $\overline{AD}$ at C. Point C is the midpoint of $\overline{BE}$.	1. Given
2. $\overline{AC} \cong \overline{DC}$	2. Definition of bisects
3. $\overline{BC} \cong \overline{EC}$	3. Definition of midpoint
4. $\angle BCA \cong \angle ECD$	4. ?
5. $\triangle ABC \cong \triangle DEC$	5. SAS triangle congruence postulate

A. Supplementary angles are congruent.

B. Corresponding angles are congruent.

C. Opposite angles are congruent.

D. Vertical angles are congruent.

4. Use the figure below. $\overline{AB} \cong \overline{DB}$ and $m\angle ABC = m\angle DBC$.

Provide the missing reasons in the proof to show that $\overline{AC} \cong \overline{DC}$.
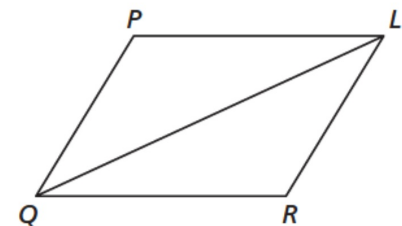
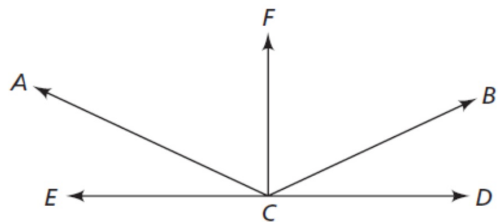| Statements	Reasons
a. $\overline{AB} \cong \overline{BD}$	Given
b. $m\angle ABC = m\angle DBC$	?
c. $\overline{BC} \cong \overline{BC}$	?
d. $\triangle ABC \cong \triangle DBC$	?
e. $\overline{AC} \cong \overline{DC}$	?

5. In this figure, $\overline{PL} \cong \overline{RQ}$ and $\overline{PQ} \cong \overline{RL}$. Prove each of the following.

a. $\triangle QPL \cong \triangle LRQ$

b. $\angle P \cong \angle R$

6. Use the figure below.

$m\angle DCB = m\angle ECA$. Points E, C, and D are collinear. $\overline{FC} \perp \overline{ED}$.

a. Based on the given information, what can you prove?

b. Prove your conjecture from part (a).