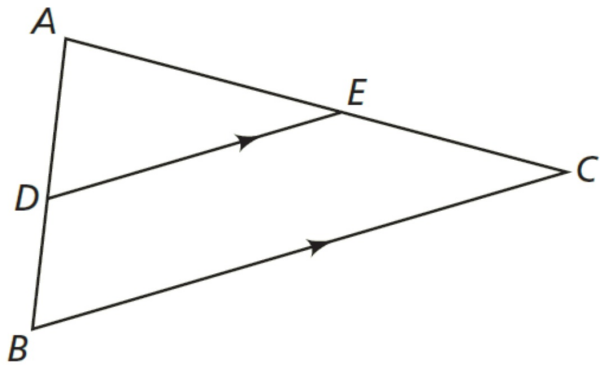


**LAUNCH:** How many triangles do you see?



## 4.10 Nested Triangles

Objectives:

- Investigate proportional relationships in nested triangles.
- Investigate how lines parallel to a side of a triangle cut the other two sides.

**nested triangles**—one triangle inside another

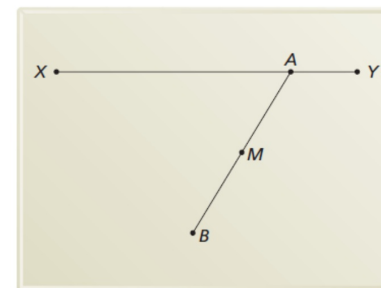
Write "IN CLASS EXPERIMENT" in your notes

Pick up a calculator and open a new Geometry document

Open your book to page 308 and follow along

### Part 1B Midpoints and Parallel Lines

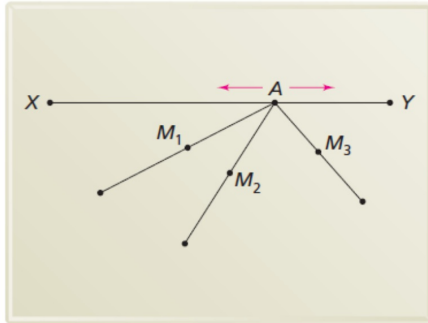
Use geometry software. Draw  $\overline{XY}$ . Construct a point  $A$  on  $\overline{XY}$  and a point  $B$  that is not on  $\overline{XY}$ . Draw  $\overline{AB}$  and construct its midpoint  $M$ . Drag point  $A$  back and forth along the entire length of  $\overline{XY}$  while tracing the path of point  $M$ .



Drag point  $A$  along  $\overline{XY}$ .

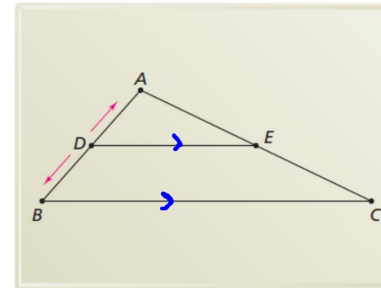
3. a. Describe the path traced by  $M$ .  
b. How does the path traced by point  $M$  compare to  $\overline{XY}$ ?
4. Repeat the construction above. This time, instead of constructing the midpoint of  $\overline{AB}$ , place the point  $M$  somewhere else on  $\overline{AB}$ . How does the position of  $M$  affect the path traced by  $M$ ?

5. Use geometry software. Draw  $\overline{XY}$ . Locate a point  $A$  on  $\overline{XY}$ . Then construct three segments, each with  $A$  as an endpoint. Construct the midpoints  $M_1$ ,  $M_2$ , and  $M_3$  of the three segments. Move point  $A$  back and forth along  $\overline{XY}$  while tracing the paths of these midpoints. Describe the paths of the midpoints, including what you know about their locations and lengths.



• **Part 2 Splitting Two Sides of a Triangle**

Use geometry software. Draw  $\triangle ABC$ . Place a point  $D$  anywhere on side  $\overline{AB}$ . Then construct a segment  $\overline{DE}$  that is parallel to  $\overline{BC}$ .



Drag point  $D$  along  $\overline{AB}$ .

6. Use the software to find the ratio  $\frac{AD}{AB}$ .
7. Find two other length ratios with the same value. Do all three ratios remain equal to each other when you drag point  $D$  along  $\overline{AB}$ ?
8. As you drag  $D$  along  $\overline{AB}$ , describe what happens to the figure. Make a conjecture about the effect of  $\overline{DE}$  being parallel to  $\overline{BC}$ .

Minds in Action: episode 13

p.310

\_\_\_\_\_ as Tony

\_\_\_\_\_ as Sasha

**Minds in Action** episode 13



*Tony and Sasha are finishing the In-Class Experiment.*

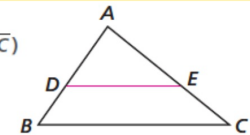
- Tony** I like dragging points on the computer screen and watching what happens.
- Sasha** Me too. Triangle  $ADE$  and the ratios all got small when we dragged  $D$  close to  $A$ .
- Tony** And when we dragged  $D$  close to  $B$ , the two triangles were almost the same, and the ratios were almost 1!
- Sasha** The two triangles always had the same shape too. I think that happened because we constructed  $\overline{DE}$  parallel to  $\overline{BC}$ .
- Tony** The parallel segment seemed to make everything work nicely.
- Sasha** So, can we make a conjecture about what having a parallel segment like  $\overline{DE}$  does for the figure?
- Tony** Can we say something like "A parallel-to-one-side segment inside a triangle makes two proportional triangles"?
- Sasha** Hmm. I get the idea. I think we have to work on the wording.

**Definitions**

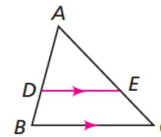
In  $\triangle ABC$  with  $D$  on  $\overline{AB}$  and  $E$  on  $\overline{AC}$ ,  $\overline{DE}$  splits two sides proportionally ( $\overline{AB}$  and  $\overline{AC}$ )

if and only if  $\frac{AB}{AD} = \frac{AC}{AE}$ .

You call the ratio  $\frac{AB}{AD}$  the **common ratio**.

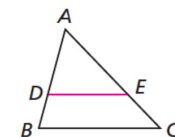


**The Parallel Side-Splitter Theorem**



$$\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$$

**The Proportional Side-Splitter Theorem**

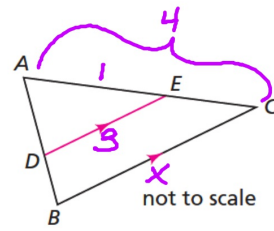


$$\frac{AB}{AD} = \frac{AC}{AE} \Rightarrow \overline{DE} \parallel \overline{BC}$$

## Examples: p.311

In Exercises 1–5,  $\overline{DE} \parallel \overline{BC}$ .

1. If  $AD = 1$ ,  $AB = 3$ , and  $AE = 2$ , what is  $AC$ ?
2. If  $AE = 4$ ,  $AC = 5$ , and  $AB = 20$ , what is  $AD$ ?
3. If  $AD = 3$ ,  $DB = 2$ , and  $AE = 12$ , what is  $EC$ ?
4. If  $AE = 1$ ,  $AC = 4$ , and  $DE = 3$ , what is  $BC$ ?
5. If  $AD = 2$  and  $DB = 6$ , what is the value of  $\frac{DE}{BC}$ ?



4.  $\frac{1}{4} = \frac{3}{x}$

$12 = x$