

5.2 Area and Perimeter

Objective: To approximate areas with inner and outer sums.

Launch:

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We need a Tony and a Sasha

Minds in Action

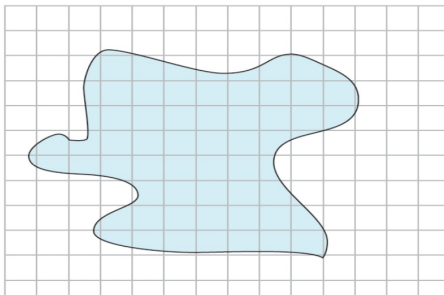
episode 15

Tony and Sasha are discussing the method for finding the area of the blob in Exercises 4 and 5.

Tony There's a pretty wide range between the inner and outer areas we found for the blob in these two exercises.

Sasha Well, the grid that the blob is on is made of really big squares. I bet if we put the blob on a grid with smaller squares, there will be less of a range. By doing that, more of the blob will be covered by whole squares.

Tony Let's try this grid. Each square is $\frac{1}{4}$ in. by $\frac{1}{4}$ in. So each square has an area of $\frac{1}{16}$ square inch.



Sasha I'll take it from here. There are 32 squares completely inside the blob. So that's $32 \times \frac{1}{16}$ square inch, or $\frac{32}{16}$ square inches. There are 47 border squares. So that's $(32 + 47) \times \frac{1}{16}$ square inch, or $\frac{79}{16}$ square inches.

Tony Hmm, the area of the blob must be somewhere in between those two numbers.

Sasha That's really not very accurate. We'd better make a finer mesh!

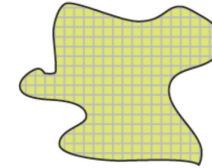
What does a finer mesh mean??

What does a finer mesh or graph paper have to do with it being more accurate??

Limit - the value a sequence of numbers gets increasingly close to.

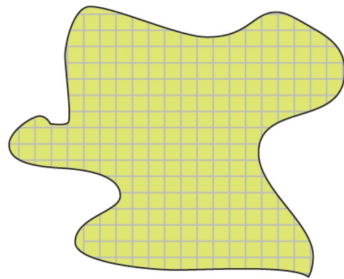
In-Class Experiment

Imagine that a blob and a grid of squares are drawn onto a big rubber sheet.



The area of these ~~228~~ squares gives a reasonably good estimate of the blob's area. **288**

Now imagine that the rubber sheet is stretched uniformly in all directions by a factor of 2. This causes the blob and the squares to be scaled by 2 as well.



Estimate the area of the blob, now that the squares have been stretched by a factor of 2.

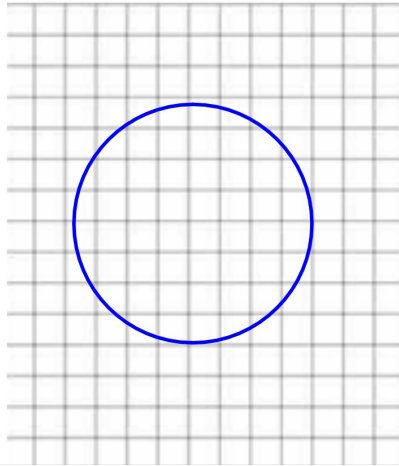
3. By what factor did the area of each square increase? **4** (6)
4. What is a good estimate for the blob's area in terms of the number of squares? **288 (same as before)**
5. If the area of one square before was S , what is it now? What is the area of the ~~228~~ stretched squares? **$4S$; $4S(288) = 1152S$**

For You to Do

6. What would have happened if you had stretched the rubber sheet uniformly in all directions by a factor of r ? Answer the questions from the In-Class Experiment for a rubber sheet stretched by a factor of r .

Check Your Understanding

1. Draw a circle of radius one inch and approximate its area using each of the following mesh sizes. Describe any patterns that show up in your estimates.
- a. $\frac{1}{2}$ in. b. $\frac{1}{4}$ in. c. $\frac{1}{8}$ in. d. $\frac{1}{16}$ in.

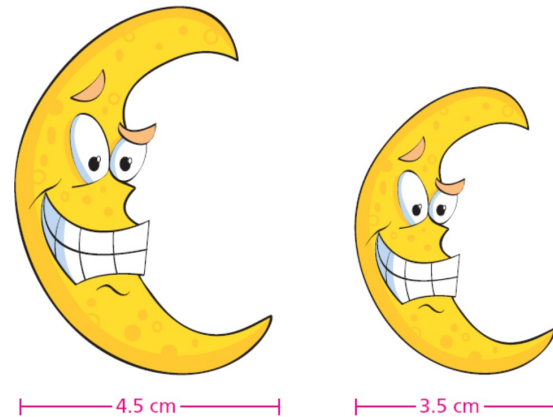


4. How do the areas of the two circles compare?
- a. A circle of radius 2 is scaled to a circle of radius 6.
b. A circle of radius 2 is scaled to a circle of radius 1.

HOMEWORK

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3. The two crescent moons are scaled copies of each other. What is the ratio of their areas?



5.2

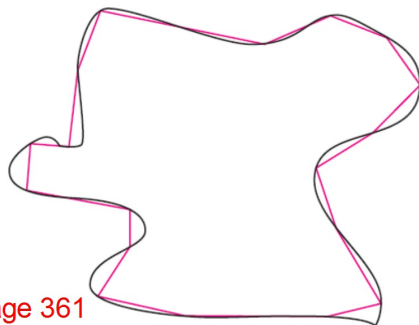
Area and Perimeter

Day 2

Goal:
Approximate perimeters of closed curves with linear approximation.

11. a. Draw a blob.
- Estimate its area using a mesh of $\frac{1}{2}$ in.
 - Estimate the same blob's area using a mesh of $\frac{1}{4}$ in.
- b. Now draw the blob again, scaled by a factor of 2. Estimate the new blob's area three times. Use a mesh of $\frac{1}{2}$ in., $\frac{1}{4}$ in., and 1 in.
- c. Which estimates in part (a) and part (b) are approximately the same?
- d. Explain the following claim. You have a good estimate for a blob's area in terms of a number of squares on graph paper. That same number of squares is a good estimate if the graph paper and the blob get stretched by a factor of r .

Archimedes used a **linear approximation** method for estimating the length of a curved path that is easy to apply. Just approximate the curve with line segments and add the lengths of the segments.



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For You to Do

7. Approximate the perimeter of the blob above using this linear approximation technique.

For Discussion

8. How can you improve your estimate?

Perimeters of Circles

The perimeter of a circle can be found through the following process.

- Inscribe a regular polygon in the circle. Circumscribe a regular polygon with the same number of sides around the circle.
- Calculate the perimeter of each polygon.
- Make another pair of inscribed and circumscribed polygons with double the number of sides. Then calculate the perimeters of the new polygons.
- Continue this process. The inner and outer perimeters will approach a common value. That number is the circle's circumference.

To **inscribe** a polygon in a circle means to draw it so that all of its vertices are on the circle.

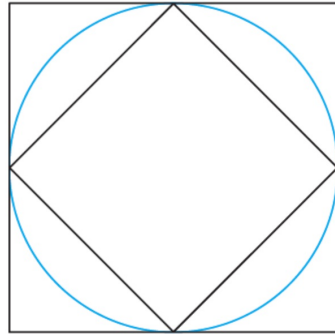
To **circumscribe** a polygon around a circle means to draw it so that all of its sides are touching the circle.

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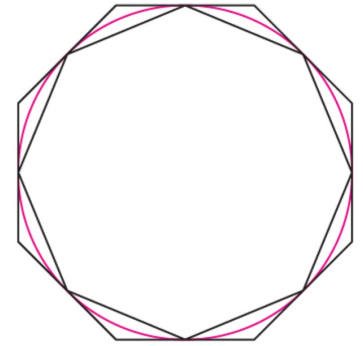
With a partner do 5 - 10

In Exercises 5–10, you will practice the process of drawing inscribed and circumscribed polygons for a circle. If you are not able to make the drawings yourself, just copy and complete the table in Exercise 8 by taking measurements directly from the drawings provided.

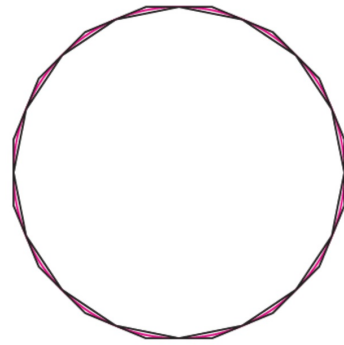
5. Draw a circle. Inscribe a square in the circle and circumscribe a square around the circle. Calculate the perimeters of the two squares and thus place the circumference of the circle between two numbers.



6. Using a circle of the same size, inscribe a regular octagon in the circle and circumscribe a regular octagon around the circle. Calculate the perimeters of the two octagons and thus place the circumference of the circle between two numbers.



7. Carry this process one step further with inscribed and circumscribed 16-gons.



8. Copy the table. Use the data from Exercises 5–7 to fill it in.

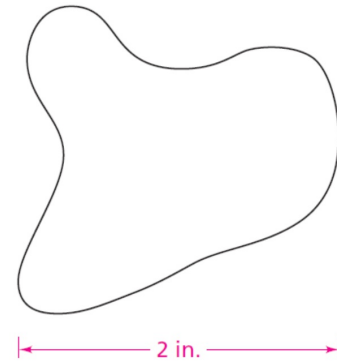
Number of Sides	Outer Perimeter	Inner Perimeter	Difference
4	■	■	■
8	■	■	■
16	■	■	■

9. Give an approximation for the perimeter of your circle.
10. Explain why the difference between the outer and inner perimeters gets smaller as the number of sides gets bigger.

On Your Own

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13. Use Sasha's regular approximation method to estimate the length of the curve below.



14. Many people use Sasha's regular approximation to estimate distance on road maps.
- Explain how this works.
 - Use a road map and regular approximation to estimate the distance between your hometown and a city many states away.

15. **Standardized Test Prep** The circle has a square circumscribed around it. It has another square inscribed inside it. Which value best describes the ratio of the perimeter of the circumscribed square to the perimeter of the inscribed square?

A. $\sqrt{2}$ B. 2 C. $2\sqrt{2}$ D. 4

