

## Launch:

Please read the following paragraph and copy Theorem 5.3 in your notes.

How does the area of a circle change when the circle is scaled? Since you know you can approximate a circle's area with a sequence of regular  $n$ -gons, you can scale all of the polygons by  $s$  to approximate the area of the scaled circle. The polygons' areas would all change by a factor of  $s^2$ , so it seems plausible that the circle's area would, too.

Below is a theorem that summarizes this.

### Theorem 5.3

If a circle is scaled by a positive number  $s$ , then its area is scaled by  $s^2$ .

## 5.5 An Area Formula for Circles

Objective: To use the definition of  $\pi$  as the area of the unit circle.

### Theorem 5.4

If the area of a circle with radius 1 is  $k$ , then the area of a circle with radius  $r$  is  $kr^2$ .

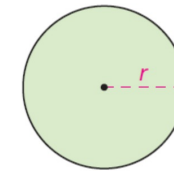
### Definition

$\pi$  is the numerical value of the area of a circle with radius 1.

$\pi$  is usually defined as the ratio of the circumference of a circle to its diameter.

### Theorem 5.5

The area of a circle of radius  $r$  is  $\pi$  times the radius squared.



$$A = \pi r^2$$

$$A = \pi(1)^2$$

$$A = \pi(1) = \pi$$

## For Discussion

6. Tony is puzzled. He asks himself, "What do they mean  $\pi$  is the area of a circle of radius 1? One what? If you have a circle of radius 1 foot, it can't have the same area as a circle whose radius is 1 inch. This is all nonsense." Suggest an answer to Tony's question.

## For You to Do

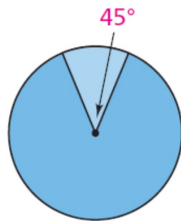
$$A = \pi r^2$$

In Exercise 1 in Lesson 5.2, you approximated the area of a circle with radius one inch to be a bit more than three square inches. Use that result and the theorem above to find a good approximation for the area of a circle with each radius.

1. 2 inches  $A = (3.14)(2^2) \approx 12.56 \text{ in}^2$     5 inches  $78.5 \text{ in}^2$   
3. 6 inches  $113.04 \text{ in}^2$     4.  $\sqrt{3}$  inches  $9.42 \text{ in}^2$   
5.  $7\frac{1}{2}$  inches  $176.625 \text{ in}^2$

$$\pi = 3.14$$

2. The angle of the wedge in the circle is  $45^\circ$ . The radius of the circle is 1.



This wedge is really called a sector of the circle, which is a region bound by two radii and the circle.

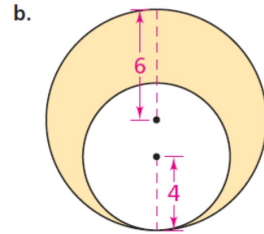
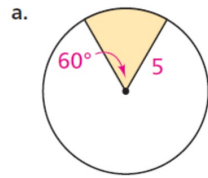
- ✓ a. What fraction of the circle's area is the wedge?  $\frac{45}{360} = \frac{1}{8}$   
✓ b. What is the exact area of the circle?  $3.14 \text{ u}^2$   
c. What is the exact area of the wedge?  $3.14/8$      $\pi/8$   
d. Use two common approximations for  $\pi$  to find the area of the wedge.

$$\approx 0.39 \text{ u}^2$$

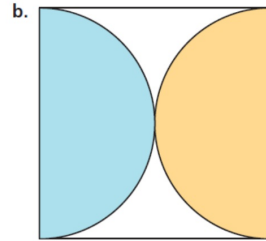
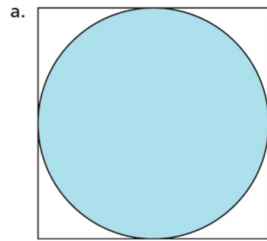
## On Your Own

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4. Find the area of each shaded region.



6. Suppose the side length of each square is 6 cm. Find the area of each shaded region and each white region.



c. Compare your answers for parts (a) and (b) above. What do you notice? Explain.

8. **Standardized Test Prep** For Zach's birthday, his grandmother makes an extra large cheesecake. She usually uses a pan that has a 7-inch diameter. This time she uses a pan with a 14-inch diameter. She usually serves wedges that are  $\frac{1}{12}$  of the cake. This time Zach asks for a wedge with an angle twice the usual size. How many times greater than normal is the area of Zach's birthday serving?

- A. 2 times      B. 4 times      C. 6 times      D. 8 times