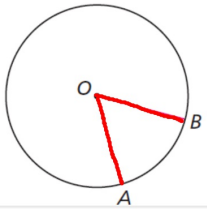
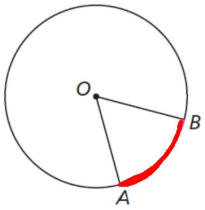
Launch: Record in your notes (p.392)

Definitions

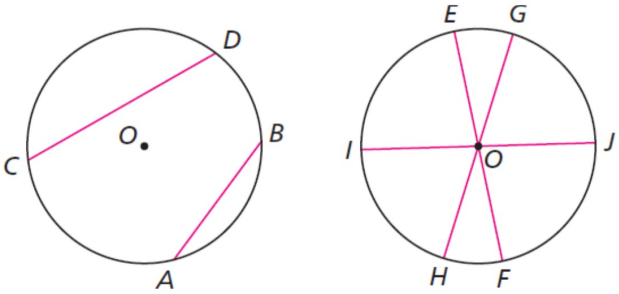
A **central angle** for a circle is an angle that has its vertex at the center of the circle.



The **arc intercepted by the angle** is the set of points on a circle that lie in the interior of a particular central angle.



A **chord** is a segment that connects two points on a circle.



A **diameter** is a chord through the center of the circle.

5.8 Arcs and Central Angles

Objective: To prove and use general theorems on chords.

Minds in Action episode 17

Looking at the definition of diameter, *Tony wonders about chords.*

Tony I once heard that the diameter is the longest chord you can draw for a given circle. Did you know that, Sasha?

Sasha As a matter of fact, I did! I think I can prove it. Let's see.

Tony Well, make life simple and start with the first circle at the bottom of the last page. You already have a chord, CD.

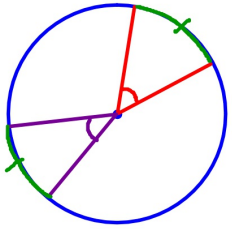
Sasha Right! All we have to do is look for triangles, and I love triangles! Connect C, D, and O. Now I remember that in a triangle the sum of two sides is always greater than the third one. So $CD < CO + OD$.

Tony You're brilliant! I know what to do now. I just noticed that CO and OD are two radii, so their sum is equal to the diameter. So we've proven that any chord CD is shorter than a diameter.

For Discussion

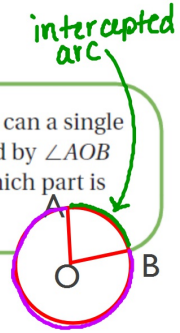
1. How does Sasha and Tony's discussion prove that the diameter is the longest chord of a circle?

If two central angles are congruent, then their intercepted arcs are congruent.



For Discussion

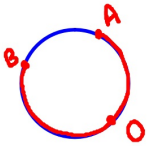
4. Tony is a bit confused by the definition of arc. He thinks, "How can a single arc be intercepted by a central angle? I see two arcs intercepted by $\angle AOB$ because the circle is divided into a big part and a little part. Which part is the intercepted arc?" How could you answer Tony's questions?



The degree measure of an arc is the measure of the central angle that intercepts it.

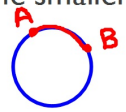
The major arc is the larger part of the circle. (greater than 180°)

$\overset{\frown}{AOB}$ (3 letters)



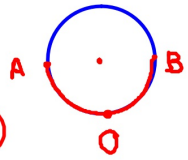
The minor arc is the smaller part of the circle. (less than 180°)

(2 letters) $\overset{\frown}{AB}$



A semicircle is half of a circle. (180°)

$\overset{\frown}{AOB}$ (3 letters)

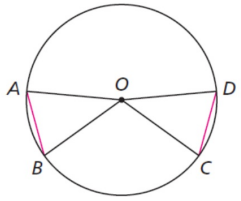


Remember...

The degree measure of an arc divided by 360 tells you how much of the circle is used by the arc.

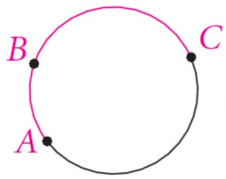
Theorem 5.7

Two chords are congruent if and only if their corresponding arcs are congruent.



$\overline{AB} \cong \overline{CD} \Leftrightarrow \overset{\frown}{AB} \cong \overset{\frown}{CD}$

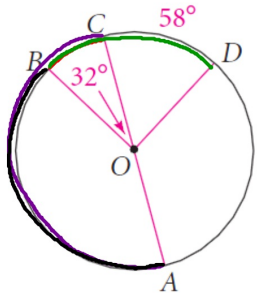
Arc Addition Postulate



$$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$$

Example

Find the measure of each arc.



a. $\overset{\frown}{BC}$ $m\overset{\frown}{BC} = 32°$

b. $\overset{\frown}{BD}$ $m\overset{\frown}{BD} = 90°$

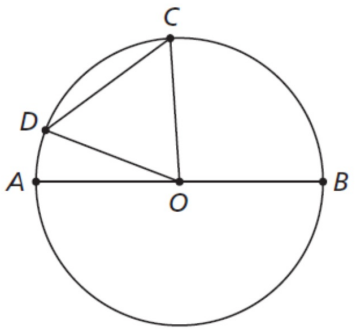
c. $\overset{\frown}{ABC}$ $m\overset{\frown}{ABC} = 180°$

d. $\overset{\frown}{AB}$ $m\overset{\frown}{AB} = 148°$

Check Your Understanding

Homework: p.395 (1,3,4)

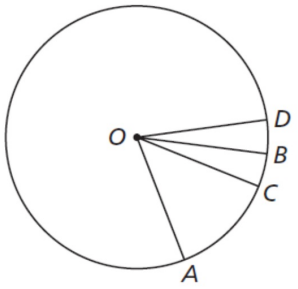
1. What parts of the circle are the following elements?



chord
diameter
radius
central angle
minor arc
major arc
semicircle

a. $\overline{AB}$
b. $\overline{OB}$
c. $\overline{OD}$
d. $\overline{CD}$
e. $\angle COD$
f. $\angle AOC$
g. $\overset{\frown}{CD}$
h. $\overset{\frown}{AB}$

3. Look at this picture, where $m\overset{\frown}{AB} = 60°$, $m\overset{\frown}{CD} = 30°$, and $m\angle AOC = 45°$.



Find the measures of the following elements. (The arcs are all minor arcs.)

a. $\angle AOB$
b. $\angle COD$
c. $\overset{\frown}{AC}$
d. $\overset{\frown}{CB}$
e. $\angle COB$
f. $\angle BOD$
g. $\overset{\frown}{AD}$

4. Is it true that if point C is on minor arc $\overset{\frown}{AB}$ of a circle with center O, then $m\overset{\frown}{AC} + m\overset{\frown}{CB} = m\overset{\frown}{AB}$? Explain.