6.2 & 6.3 Geometric and Arithmetic Means

Objective: To find and use relationships in similar right triangles.

Geometric mean of two positive numbers a and b is:

$$\frac{a}{x} = \frac{x}{b}$$

$$x = \sqrt{ab}$$

	x^2
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225

Example 1
Find the geometric mean of:

a) 4 and 18

$X = \sqrt{4 \cdot 18}$
$X = \sqrt{72}$
$X = \sqrt{36 \cdot 2}$
$X = 6\sqrt{2}$

b) 15 and 20

$X = \sqrt{20 \cdot 15}$
$X = \sqrt{300}$
$X = \sqrt{100 \cdot 3}$
$X = 10\sqrt{3}$

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arithmetic mean, or average of two numbers a and b is

$$\frac{a + b}{2}$$

Example 2
Find the **arithmetic** mean of:

a) 4 and 18

$\frac{4 + 18}{2} = \frac{22}{2} = 11$

b) 15 and 20

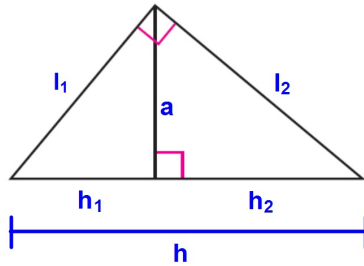$\frac{15 + 20}{2} = \frac{35}{2} = 17.5$

What do you notice about the geometric mean compared to the arithmetic mean?

The Arithmetic-Geometric Inequality

arithmetic mean ≥ geometric mean

$$\frac{a + b}{2} \geq \sqrt{ab}$$

The altitude divides the triangle into 3 similar triangles.

$$a = \sqrt{h_1 \bullet h_2}$$ The altitude is the geometric mean of h_1 and h_2
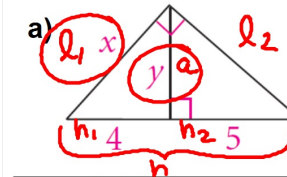
$$l_1 = \sqrt{h_1 \bullet h}$$ The short leg is the geometric mean of h_1 and h

$$l_2 = \sqrt{h_2 \bullet h}$$ The longer leg is the geometric mean of h_2 and h

Example 3
Solve for x and y.

a)

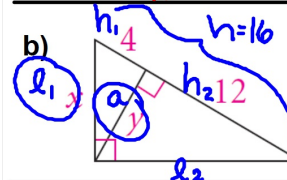$l_1 : \sqrt{h_1 \cdot h}$
$x = \sqrt{4 \cdot 9}$
$x = \sqrt{36}$
$\boxed{x = 6}$

$a = \sqrt{h_1 \cdot h_2}$
$y = \sqrt{4 \cdot 5}$
$y = \sqrt{20}$
$y = \sqrt{4 \cdot 5}$
$\boxed{y = 2\sqrt{5}}$

b)
$h = 16$

$l_1 : \sqrt{h_1 \cdot h}$
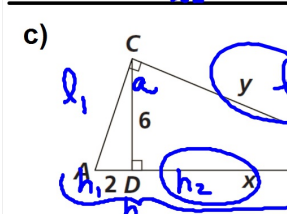$x = \sqrt{4 \cdot 16}$
$x = \sqrt{64}$
$\boxed{x = 8}$

$a = \sqrt{h_1 \cdot h_2}$
$y = \sqrt{4 \cdot 12}$
$y = \sqrt{48}$

$\sqrt{16 \cdot 3}$
$\boxed{4\sqrt{3}}$

c)

$a = \sqrt{h_1 \cdot h_2}$
$l_2 = \sqrt{h_2 \cdot h}$
$6 = \sqrt{2x}$
$6^2 = \sqrt{2x}^2$
$36 = 2x$

$\boxed{x = 18}$
$l_2 = \sqrt{h_2 \cdot h}$
$y = \sqrt{18 \cdot 20}$
$y = \sqrt{360}$

$\sqrt{36}$
$6\sqrt{10}$

Example 4

Recreation The 300-m path to the information center and the 400-m path to the canoe rental dock meet at a right angle at the parking lot. Marla walks straight from the parking lot to the lake as shown. How far is Marla from the information center?

Parking
300 m
400 m
Marla is here.
Canoe Rental