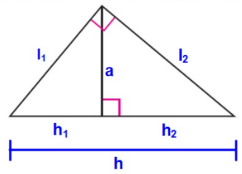
Launch:

Altitude to the Hypotenuse



$$a = \sqrt{h_1 \bullet h_2}$$

$$l_1 = \sqrt{h_1 \bullet h}$$

$$l_2 = \sqrt{h_2 \bullet h}$$

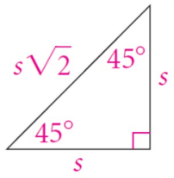
Soh-Cah-Toa

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

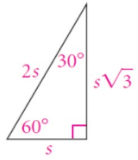
$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

45-45-90 Triangle



30-60-90 Triangle



Geometric Mean

$x = \sqrt{ab}$

Arithmetic Mean

$\frac{a+b}{2}$

6.8 Finding Triangle Areas

Objective: To use the sine, cosine, and tangent functions and their inverses to find missing side lengths and angle measures in triangles.

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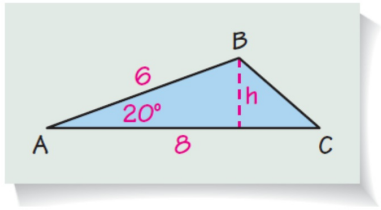
Minds in Action episode 26

Derman has made a discovery!

Derman I can find the area of any triangle if you tell me two sides and the included angle!

Tony Any triangle? Really? Okay, in $\triangle ABC$, $AB = 6$ cm, $AC = 8$ cm, and $m\angle B = 20°$.

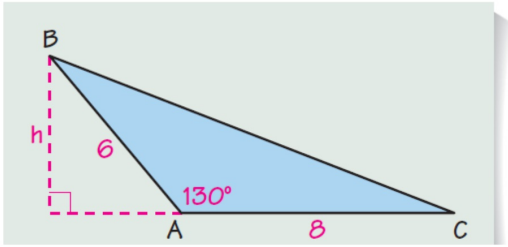
Derman Here's my sketch.



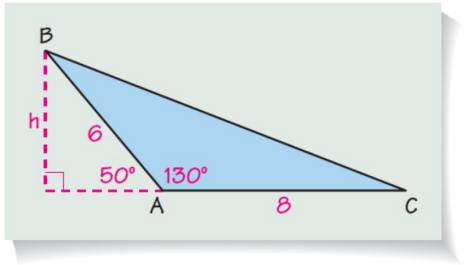
So $\frac{h}{6} = \sin 20°$ and $h = 6 \sin 20°$, which is about 2 cm. That means the area of $\triangle ABC$ is about $\frac{1}{2}(2)(8)$, or about 8 cm^2.

Tony Nice! Does it work if the angle is obtuse? After all, we've only defined the sine function for acute angles.

Derman I'm not sure. Let's try one.



Tony I see how to do it. You just use the little right triangle that has the height as one of its legs.

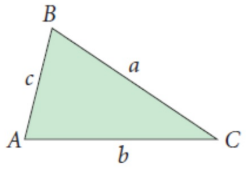


And we've got it. The area is approximately 36.8 cm^2.

Theorem 9-1 **Area of a Triangle Given SAS**

The area of a triangle is one half the product of the lengths of two sides and the sine of the included angle.

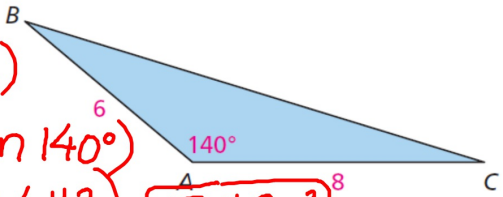
Area of $\triangle ABC = \frac{1}{2}bc(\sin A)$



For You to Do

Find the area of each triangle.

1.

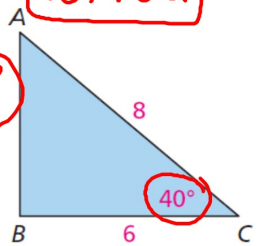


$\frac{1}{2}bc(\sin A)$

$\frac{1}{2}(6)(8)(\sin 140°)$

$\frac{1}{2}(6)(8)(0.643) = 15.43u^2$

2.

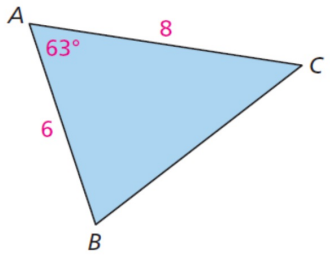


$\frac{1}{2}(6)(8)(\sin 40°)$

$\frac{1}{2}(6)(8)(0.643)$

$15.43u^2$

3.



$A = \frac{1}{2}(8)(6)(\sin 63°)$

$A = 21.38u^2$

On Your Own

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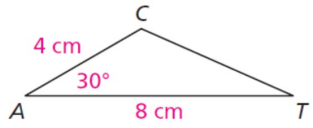
****Change in Directions****

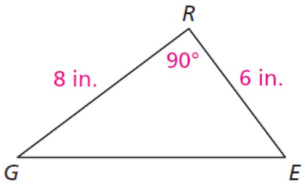
Find the area of the triangle!

In Exercises 12–15, find the missing side length and angle measures of each triangle.

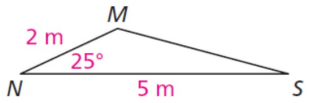
12.



13.



14.



15.