Launch:

Assume G(3,2) and B(-3,-4)

1. What are the coordinates of the midpoint of GB?

(0,-1)

2. What is the length of GB?

3. Do you think you could do this problem without graphing?

$a^2+b^2=c^2$

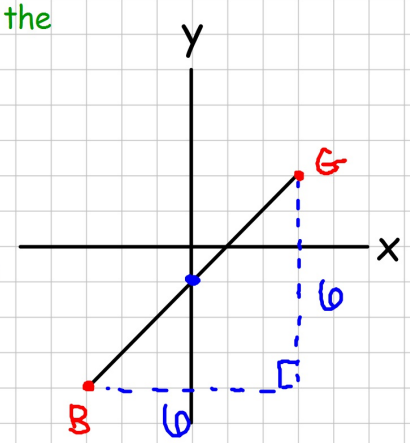
$6^2+6^2=c^2$

$36+36=c^2$

$72=c^2$

$\sqrt{72}=c$

$\sqrt{36\cdot 2}=6\sqrt{2}$



7.6 Midpoint and Distance Formulas

Objectives:

To calculate the distance between two points with given coordinates.

To calculate the coordinates of the midpoint of a segment when you know the coordinates of the endpoints of the segment.

Minds in Action episode 32

Hannah and Darren are trying to write a formula for finding the distance between two points. They are given

$$G = (x_1, y_1),\ H = (x_2, y_2)$$

Hannah Well, the distance between two points on a vertical or horizontal line is easy to find. Just subtract the unlike coordinates. Say G is $(3, 4)$ and H is $(3, 90)$. Then the distance between G and H is $90 - 4 = 86$.

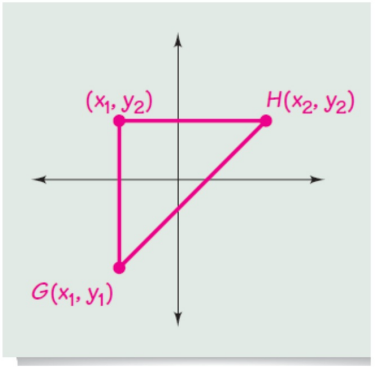
Darren But we don't know that G and H are on a horizontal or vertical line. They're just *any* two points.

We used the Pythagorean Theorem to help us find the distance between two points that weren't on the same horizontal or vertical line before. Let's try that here.

Hannah Don't we need three points to make a triangle so we can use the Pythagorean Theorem?

Darren Watch! I'll make a third point:

Before you ask, I know that the third point is (x_1, y_2). In my picture I had to go over as far as the (x_1, y_1) point—that's where I got the x_1—and up as far as the (x_2, y_2) point—that's where I got the y_2.



Hannah Great! Now let's use the Pythagorean Theorem to find the length of the hypotenuse of that triangle.

$$(y_2 - y_1)^2 + (x_2 - x_1)^2 = GH^2$$

So,

$$GH = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Theorem 7.2 Distance Formula

The distance between two points (x_1, y_1) and (x_2, y_2) can be found using the Pythagorean Theorem. It is the square root of the sum of the square of the difference in the x-coordinates and the square of the difference in the y-coordinates.

Distance Formula

$$\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

PEMDAS

Distance Formula

$$\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

For You to Do

Find the distance between each pair of points.

7. $(1, 1)$ and $(-1, -1)$
8. $(1, 1)$ and $(4, 5)$
9. $(2, 4)$ and $(-4, -2)$

$x_1 y_1$ $x_2 y_2$

8. $\sqrt{(5-1)^2+(4-1)^2}$
$\sqrt{4^2+3^2}$
$\sqrt{16+9} = \sqrt{25} = 5$

7. $\sqrt{(-1-1)^2+(-1-1)^2}$
$\sqrt{(-2)^2+(-2)^2}$
$\sqrt{4+4} = \sqrt{8}$
$\sqrt{4 \cdot 2} = 2\sqrt{2}$

9. $\sqrt{(-2-4)^2+(-4-2)^2}$
$\sqrt{(-6)^2+(-6)^2}$
$\sqrt{36+36} = \sqrt{72}$
$\sqrt{36 \cdot 2} = 6\sqrt{2}$

Theorem 7.3 Midpoint Formula

Each coordinate of the midpoint of a line segment is equal to the average of the corresponding coordinates of the endpoints of the line segment.

Midpoint Formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Midpoint Formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

For You to Do

10. Find the midpoint of the segment with endpoints $(1327, 94)$ and $(-668, 17)$.
11. Find the midpoint of the segment with endpoints $(1776, 13)$ and $(2000, 50)$.

10. $\left(\frac{1327+(-668)}{2}, \frac{94+17}{2}\right)$

11. $(329.5, 55.5)$
$(1888, 31.5)$

Homework: Worksheet Midpoint/Distance

1-20

1-8 Midpoint

9-20 Midpoint & distance