

Launch:

What is the formula we used in Algebra 1 to find the **slope** of a line when we have two points (x_1, y_1) and (x_2, y_2) ?

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

How do we find the **slope** when we have an equation of a line?

$$y = mx + b$$

↑
slope

write in slope-intercept form!

7.7 Parallel Lines and Collinear Points

Objective: To prove algebraically that three points are or are not collinear. To find and compare coefficients in equations of lines and recognize when two lines are parallel.

Theorem 7.4

Two lines are parallel if and only if they have the same slope.

Example

Are the following two lines parallel?

$$y = 5x + 6$$

$$-18x + 3y = -54$$

$$+18x \quad +18x$$

$$3y = 18x - 54$$

$$\div 3 \quad \div 3$$

$$y = 6x - 18$$

$$m = 5$$

$$m = 6$$

Not parallel

$$y = mx + b$$

Theorem 7.5

Let $A, B,$ and C be three points, no two of which are in line vertically. Points $A, B,$ and C are collinear if and only if the slope between A and $B,$ $m(A, B),$ is the same as the slope between B and $C, m(B, C).$ In symbols:

$$* A, B, \text{ and } C \text{ are collinear} \Leftrightarrow m(A, B) = m(B, C). *$$

Example

Are the three points collinear?

$$A(1, 5) \quad B(0, 1) \quad C(3, 13)$$

(Hint: we have to find the slope between each pair of points)

$$m(A, B) = \frac{1-5}{0-1} = \frac{-4}{-1} = 4$$

$$m(B, C) = \frac{13-1}{3-0} = \frac{12}{3} = 4$$

$A, B,$ & C
are
collinear

For You to Do

Use the triangle inequality or Theorem 7.5 to decide whether the three points are collinear. Use the other method to check your result.

6. $A\left(\frac{2}{7}, 0\right)$ $B(0, 2)$ $C(1, 9)$
7. $A\left(\frac{1}{25}, 2\right)$ $B(1, 26)$ $C(7, 0)$
8. $A(2, 2)$ $B(3, 3)$ $C(5, 9)$ **No**
9. $A(2, 4)$ $B(0, 0)$ $C(3, 6)$ **Yes**
10. $A(-1, 3)$ $B(-2, 2)$ $C(2, 6)$ **Yes**

1. Are the two lines with the given equations parallel? Explain.

- a. $2x + 3y = 0$ $2x + 3y = 4$
- b. $x + \frac{1}{3}y = 7$ $3x + y = 1$
- c. $x + \frac{1}{3}y = 7$ $3x + y = 21$
- d. $\sqrt{2}x + 4y = 0$ $2x + 4y = 0$
- e. $2\sqrt{2}x + 4y = 1$ $x + \sqrt{2}y = 3$