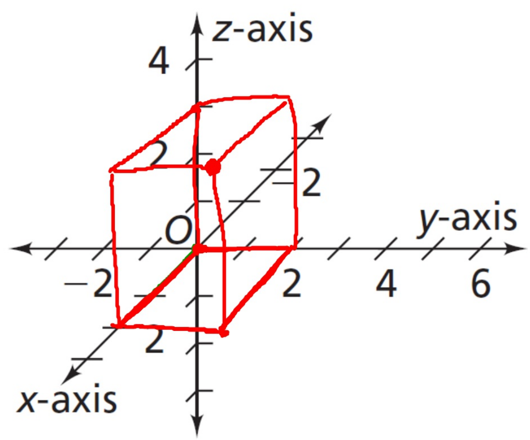
Launch: Below is a sketch of the 3D coordinate system. There is a third axis, called the z-axis.

How do you think you would graph the following point (2,2,3)? TRY IT!

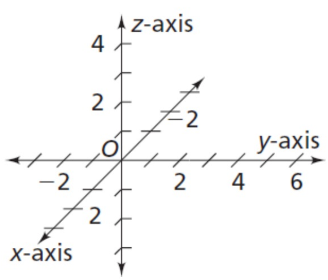
(x,y,z)



7.9 Coordinates in Three Dimensions

Objective: To plot points in three dimensions and find the distance between them.

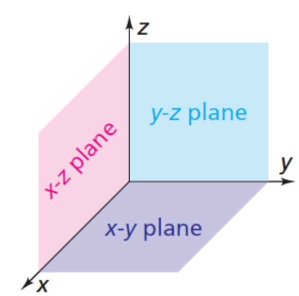
In three-dimensional space, you describe the location of a point with an ordered triple (x,y,z)



Developing Habits of Mind

Visualize. It is difficult to picture a three-dimensional figure, such as a plane, on a two-dimensional sheet of paper.

The diagram at the right might help you visualize three different planes in space. It looks like a corner of a room with no windows.



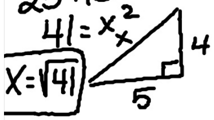
The x-y plane is the floor. All ordered triples that lie on this plane have a z-coordinate of 0: $(1, 2, 0)$, $(3, 5, 0)$, $(8, -2, 0)$, and so on.

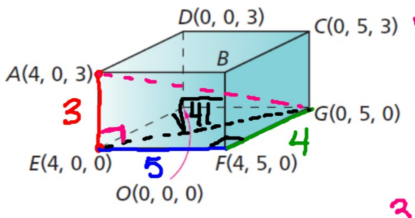
The y-z plane is the wall at the back. All ordered triples that lie on this plane have an x-coordinate of 0: $(0, 1, 2)$, $(0, 3, 5)$, $(0, -2, 8)$, and so on.

The x-z plane is the wall at the left. All ordered triples that lie on this plane have a y-coordinate of 0: $(3, 0, 2)$, $(1, 0, 5)$, $(-12, -2, 0)$, and so on.

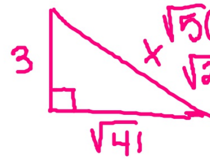
For the diagram below, Tony had to find these lengths: *AE*, *EF*, *GF*, *EG*, ~~*AK*~~ and *AG*.

$a^2+b^2=c^2$ $\to$ $5^2+4^2=x^2$ $\to$ $25+16=x^2$ $\to$ $41=x^2$ $\to$ $x=\sqrt{41}$





$a^2+b^2=c^2$ $\to$ $\sqrt{41}^2+3^2=x^2$ $\to$ $41+9=x^2$ $\to$ $50=x^2$ $\to$ $\sqrt{50}=x$ $\to$ $\sqrt{25\cdot 2}=x$ $\to$ $x=5\sqrt{2}$



Here is Tony's work.

$AE = 3$ because $\overline{AE}$ is parallel to the *z*-axis.

$EF = 5$ because $\overline{EF}$ is parallel to the *y*-axis.

$GF = 4$ because $\overline{GF}$ is parallel to the *x*-axis.

$EG = \sqrt{41}$ because of the Pythagorean Theorem.

$AG = \sqrt{50}$ because of the Pythagorean Theorem. $5\sqrt{2}$

Distance Formula (3D)

Tony's work leads to a formula for the distance between two points in three dimensions.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Example:

Find the distance between (4,0,3) (0,5,0)

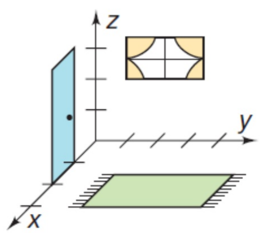
$\sqrt{(0-4)^2+(5-0)^2+(0-3)^2}$

$\sqrt{(-4)^2+(5)^2+(-3)^2}$

$\sqrt{16+25+9} = \sqrt{50} = 5\sqrt{2}$

Check Your Understanding

1. Imagine that this room in your home is on a three-dimensional coordinate system.



a. Describe where the origin is.

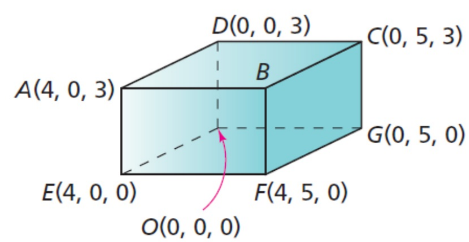
Estimate the ordered triples that describe each of the following.

b. the four corners of the door

c. the four corners of the window

d. the four corners of the rug

2. Here is the picture that Tony was working with earlier. Find the coordinates of the midpoint for each segment with the endpoints listed.



a. $A(4, 0, 3)$ and $C(0, 5, 3)$

b. $A(4, 0, 3)$ and $E(4, 0, 0)$

c. $O(0, 0, 0)$ and $A(4, 0, 3)$

d. $A(4, 0, 3)$ and $G(0, 5, 0)$

On Your Own

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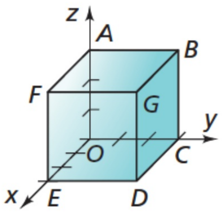
Use the diagram for Exercises 7–9.



7. Find the coordinates of each vertex of the box.

8. Find the length of each segment.

a. $\overline{OE}$ **b.** $\overline{OB}$

c. $\overline{AE}$ **d.** $\overline{FC}$

9. Name another segment with the same length as the given segment.

a. $\overline{OE}$ **b.** $\overline{OB}$

c. $\overline{AE}$ **d.** $\overline{FC}$

10. A cube has a vertex at the origin and sides of length 1 along the axes.

a. What are the other vertices of the cube?

b. What is the length of a diagonal of the cube?

13. **Standardized Test Prep** A dowel is a long, straight rod with a small diameter. You want to ship a dowel in a rectangular box. The dimensions of four boxes are listed. Which dimensions allow for the longest dowel if the dowel extends from one corner to the opposite corner of the box?

A. 44 inches by 24 inches by 12 inches

B. 40 inches by 30 inches by 4 inches

C. 35 inches by 35 inches by 2 inches

D. 27 inches by 27 inches by 27 inches