

8.5 Maximizing Areas, Part 1

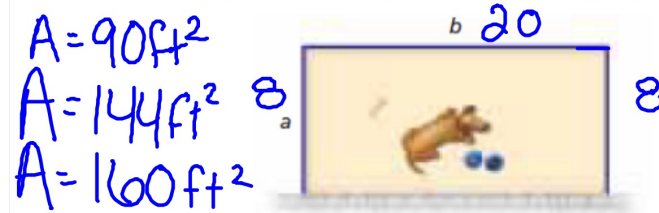
Objective:

- Maximize area for a triangle or rectangle under given conditions.
- Find the maximum area for a shape with a given perimeter.

Example

Problem You want to fence in a rectangular exercise run for your dog. You have 36 feet of fencing material. You decide that to enclose more area, you will use a wall of your house as the fourth side of the run. What are the dimensions of the greatest area you can enclose?

Solution This figure shows a run with width a and length b .



**Try values for a & b that use 36 total feet of fence.
What is the area of your dog run?
Did your neighbor have a larger area?**

a) How do you find perimeter?

$\text{Add all side lengths}$

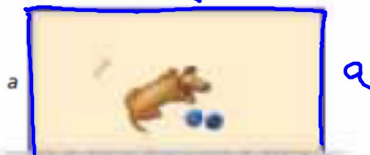
b) How much fencing do you have?

36ft.

c) What is the equation for the perimeter using a & b ?

$$\underline{a} + \underline{b} + \underline{a} = 36$$

$$2a + b = 36$$

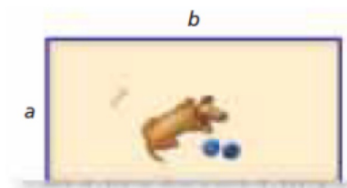


d) How do you find area of a rectangle?

$$A = bh \quad A = lw$$

e) What is the equation for the area, using a & b ?

$$A = ab$$



So we have 2 equations....

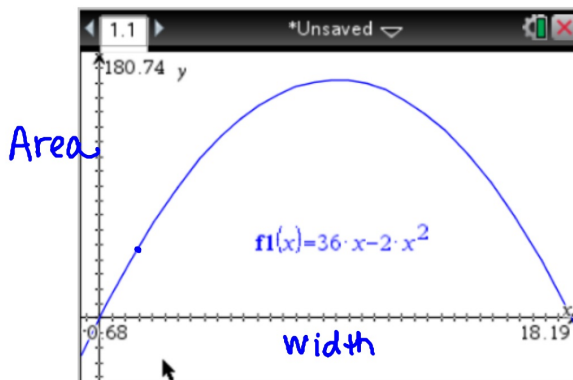
$$\begin{array}{r}
 2a + b = 36 \text{ (Perimeter)} \\
 -2a \quad -2a \\
 \hline
 b = 36 - 2a \\
 A = ab \quad \text{(Area)}
 \end{array}$$

We can rewrite the formula for area using only one variable (say a) if we use the perimeter equation.

$$\begin{array}{l}
 A = a(36 - 2a) \\
 = 36a - 2a^2
 \end{array}$$

Each **x value** represents the **width** of our fenced in plot, each **y value** represents the output for the **area**.

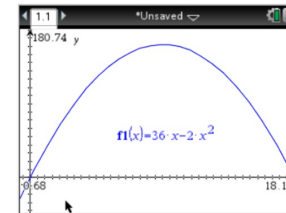
So, how can we find the width with the maximum area?



So, $A = 36a - 2a^2$. Let's graph this on our calculators.

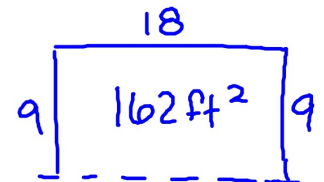
1. Menu > Add Graphs.
2. Type in $f1(x) = 36x - 2x^2$
3. Press Enter.
4. We need to zoom out, the graph is too large. Press Menu>Window/Zoom>Window Settings
5. Now, you should have the following:

Sketch your graph!



Solution: The highest point on the graph should tell us both the width for the maximum area and the actual area.

1. Menu>Analyze Graph> Maximum
2. Pick a lower and upper bound...

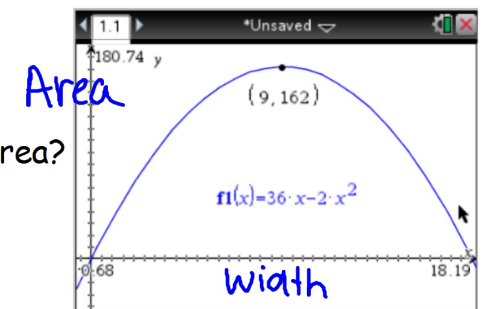


What is the width?

9 ft.

What is the maximum area?

162 ft²

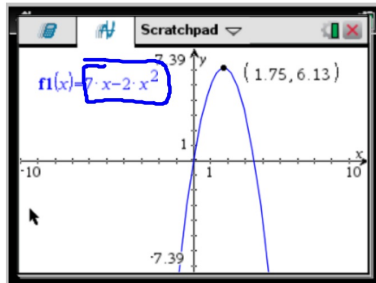


For you to do:

1. Verify that this is indeed the maximum area. Find the area of the plot when the width is 8 and 10. Are they less than 162?

8 $\boxed{8}$ 8 $A=160\text{ft}^2$ 10 $\boxed{10}$ 10 $A=160\text{ft}^2$

2. Now, you're making a maximum enclosure for a pet rabbit instead of a dog. You've got 7 feet of fence. What is the maximum area you can enclose?



Answer: 6.13 ft²

SUMMARY/FOR DISCUSSION:

Can you write a formula that would work for any scenario involving the fence?

For example...

What if you had a horse and 350 feet of fence?

A heard of cattle and 5,000 feet of fence?